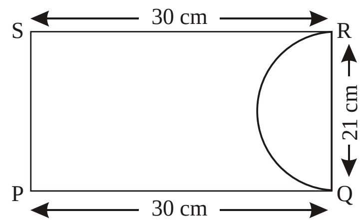
11 Area of Plane Figures

Exercise 11.1



1. $PQ = 30\,cm$; $QR = 21\,cm$

Area of rectangle $PQRS = PQ \times QR$

$= 30 \times 21\,cm^2$

$= 630\,cm^2$

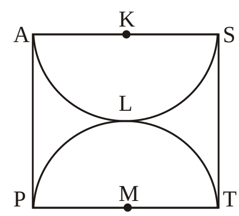
Area of Semicircle

$= \frac{1}{2}\pi r^2 \qquad \left(\therefore\ r = \frac{21}{2}cm\right)$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} cm^2$

$= 173.25\ cm^2$

Area of the remaining part $= (630 - 173.25)\,cm^2 = 456.75\,cm^2$



2. Side of the square $PAST = 14\,cm$

$AS = 14\,cm$; $PT = 14\,cm$

ALS ; Semicircle $= \frac{1}{2}\pi r^2 \qquad \left(\therefore\ r = \frac{14}{2} = 7\,cm\right)$

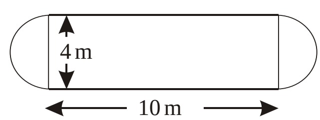
$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 cm^2 = 77\,cm^2$

PLT ; Semicircle $= \frac{1}{2}\pi r^2 \qquad \left(\therefore\ r = \frac{14}{2} = 7\,cm\right)$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\,cm^2 = 77\,cm^2$

Area of square $PAST$ = square of side $= (14)^2\,cm^2 = 196 cm^2$

Area of the shaded part $= (196 - 77 - 77)\,cm^2 = 196 - 154\ cm^2 = 42\,cm^2$



3. Diameter of circular $= 4\,m$

Radius of circle $r = \frac{4}{2}\,m = 2\,m$

Length of the rectangle $(l) = 10\,m$

Breadth of the rectangle $(b) = 4\,m$

Area of rectangular part $= l \times b$

$= 10 \times 4\,m^2 = 40\,m^2$

Area of semicircular end $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 2 \times 2 = 6.285\,m^2$

Area of semicircular end $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 2 \times 2 = 6.285\,m^2$

Area of the play ground $= (40 + 6.29 + 6.29)\,m^2 = 52.57\,m^2$

4. Let, length of rectangular field $= x$ cm

After increase 50% length of rectangular

$$x + 50\% \text{ of } x = x + \frac{x}{2} = \frac{3x}{2} \text{ cm}$$

Let breadth $= y$ cm

Decrease 50% breadth of rectangular field

$$y - \frac{y}{2} = \frac{1y}{2} = \frac{y}{2}$$

$$\text{Area of field } = xy \text{cm}^2$$

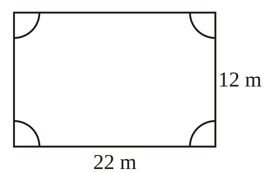
$$\text{Area of field after change length and breadth } = \frac{3x}{2} \times \frac{y}{2} = \frac{3xy}{4} \text{cm}^2$$

$$\text{change in area } = xy - \frac{3xy}{4} = \frac{xy}{4} \text{ cm}^2$$

$$\% \text{ of change area } = \frac{xy/4}{xy} \times 100 = 25\%$$

So, decrease in area by 25%.

5.



$$\text{Length of rectangle } = 22 \text{ m}$$

$$\text{Wide of rectangle } = 12 \text{ m}$$

$$\text{Area of rectangle } = \text{length} \times \text{wide}$$

$$= (22 \times 12) \text{ m}^2$$

$$= 264 \text{ m}^2$$

$$\text{Area of circle } = \pi r^2$$

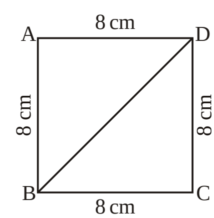
$$= \frac{22}{7} \times 2.5 \times 2.5 \text{ m}^2$$

$$= 19.64 \text{ m}^2$$

$$\text{Area of the remaining part } = (264 - 19.64) \text{ m}^2$$

$$= 244.36 \text{ m}^2 \text{ or } 244\frac{5}{14} \text{ m}^2$$

6. Square of $ABCD$



ΔABC,

$AB = 8$ cm, $CB = 8$ cm

$$(AC)^2 = (BA)^2 + (CB)^2$$

$$(AC)^2 = (8) + (8)^2$$

$$= 64 + 64 = 128$$

$$AC = \sqrt{128} = 8\sqrt{2}$$

length of diagonal $= 8\sqrt{2}$ cm

7.

$$\text{Let sides of one square } = 4x \text{ cm}$$

$$\text{Side of second square } = 5x \text{ cm}$$

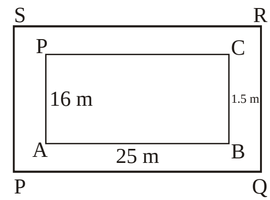
$$\text{Area of one square } = 4x \times 4x = 16x^2 \text{ cm}^2$$

$$\text{second square } = 5x \times 5x = 25x^2 \text{ cm}^2$$

$$\text{Ratio of both square } = \frac{16x^2}{25x^2}$$

$$= \frac{16}{25} = 16:25$$

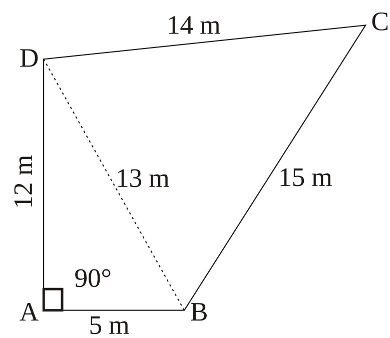
8.



$$\text{Area of } ABCD = 25 \times 16\,\text{m}^2$$
$$= 400\,\text{m}^2$$
$$\text{Area of } PQRS = (25 + 1.5 \times 2) \times (16 + 1.5 \times 2)\,\text{m}^2$$
$$= 28 \times 19\,\text{m}^2$$
$$= 532\,\text{m}^2$$
$$\text{Area of footpath} = PQRS - ABCD$$
$$= (532 - 400)\,\text{m}^2 = 132\,\text{m}^2$$
$$\text{Area of footpath} = 132 \times 10000\,\text{cm}^2 = 1320000\,\text{cm}^2$$
$$\text{Tile measuring} = 20\text{cm} \times 20\,\text{cm}$$
$$\text{Area of Tile} = 400\text{cm}^2$$
$$\text{No of tiles covered in footpath} = \frac{1320000\text{cm}^2}{400\text{cm}^2}$$
$$= 3300$$

9. Join the vertex B and D.

Now, in ΔABD, by pythagoras theorem



$$BD = \sqrt{AB^2 + AD^2}$$
$$= \sqrt{5^{2+}\ 12^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169} = 13\,\text{m}$$

So, $$\text{the area of } ABD = \frac{1}{2} AB \times AD$$
$$= \frac{1}{2} \times 5 \times 12 = 5 \times 6 = 30\,\text{m}^2$$

In ΔBDC,

It is a scalene triangle

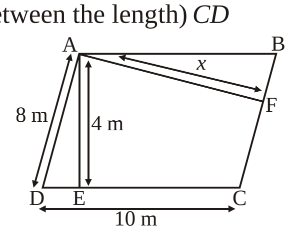
$\therefore$ $$S = \frac{BD + DC + `BC}{2} = \frac{13 + 14 + 15}{2} = \frac{42}{2} = 21\text{m}$$

$\therefore$ $$\text{the area of } \Delta BDC = \sqrt{S\ (S - BD)\,(S - DC)\,(S - BC)}$$
$$= \sqrt{21\,(21 - 13)\,(21 - 14)\,(21 - 15)}$$
$$= \sqrt{21 \times 6 \times 7 \times 8}$$
$$= \sqrt{7056} = 84\,\text{m}^2$$

Therefore, the area of a quadrilateral $=$ area of $\Delta ABD +$ area of ΔBDC
$$= (30 + 84)\,\text{m}^2 = 114\,\text{m}^2$$

10. Suppose, distance between the shorter sides $= x$ m

Then, Area of parallelogram $= AE$ (Distance between the length) CD
$$= 4 \times 10$$
$$= 40\,\text{m}^2$$



$$\text{Area of parallelogram} = 40\,\text{m}^2$$
$\because$ $$\text{Area of parallelogram} = AF \times BC$$
$$\text{So, } AF \times BC = 40\,\text{m}^2$$

$$x \times 8\ m = 40\ \text{m}^2$$

$$x = \frac{40\ \text{m}^2}{8\ \text{m}} = 5\ \text{m}$$

Hence, the distance between shorter sides is 5 m.

Exercise 11.2

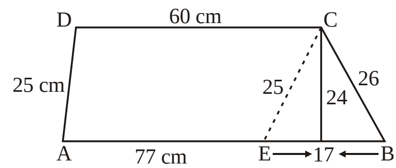
1. (a) Base = 15 cm and 20 cm and altitude $= 8\,\text{cm}$

$$\text{Area} = \frac{1}{2}(15+20)\times 8\,\text{cm}^2 = 140\,\text{cm}^2$$

(b) Base $= 10\,\text{cm}$ and 12 cm and altitude $= 5\,\text{cm}$

$$\text{Area} = \frac{1}{2}(10+12)\times 5\,\text{cm}^2 = 55\,\text{cm}^2$$

2. Draw $CE \parallel CB$



$ABCE$ is a parallelogram

$$EC = AD = 25\,\text{cm}$$

$$BE = 77 - 60 = 17\,\text{cm}$$

For ΔCEB

$$S = \frac{25+26+17}{2} = \frac{68}{2} = 34\ \text{cm}$$

$$\text{Area of } \Delta BCE = \sqrt{S\,(S-a)(S-b)(S-C)}$$

$$= \sqrt{34\,(34-17)\,(34-26)\,(34-25)}$$

$$= \sqrt{34\times 17\times 8\times 9}$$

$$= \sqrt{41616} = 204\ \text{cm}$$

$$\Delta\ \text{Area} = \frac{1}{2}\times BE \times h$$

$$= \frac{1}{2}\times 17\times h = 204$$

$$h = \frac{204\times 2}{17} = 24\,\text{cm}$$

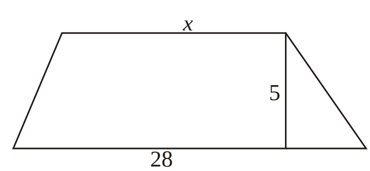
$$h = 24\,\text{cm}$$

$$\text{Area of } ABCD = \frac{1}{2}(a+b)\times h$$

$$= \frac{1}{2}\times(60+77)\times 24\,\text{cm}^2$$

$$= \frac{1}{2}\times 137\times 24\ \text{cm}^2 \qquad = 1644\ \text{cm}^2$$

3. Area of a trapezium $= 105\text{cm}^2$



Let, other side of the parallel sides is x

One side of the parallel side $= 28\text{cm}$

$$\text{Area of a trapezium} = \frac{1}{2}\times(a+b)\times h$$

$$105 = \frac{1}{2}\times(28+x)\times 5$$

$$105\times 2 = (28+x)5$$

$$210 = 140 + 5x$$

$$\frac{210-140}{5} = x$$

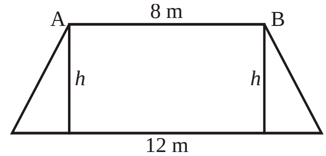
$$x = 14 \text{ cm}$$

$$\text{other side of trapezium} = 14 \text{ cm}$$

4. Suppose depth $= h$

parallel sides $= 8\text{ m}$ and 12 m

Area of trapezium $= 84\text{ m}^2$



Then area of trapezium $= \frac{1}{2} \times$ (sum of length of parallel sides) $\times$ depth.

$$84\text{ m}^2 = \frac{1}{2} \times (8+12)\text{ m} \times h$$

$$h = \frac{84 \times 2\text{ m}^2}{20\text{ m}} = 8.4\text{ m}$$

Hence, depth of the cross section of a canal is 8.4 m.

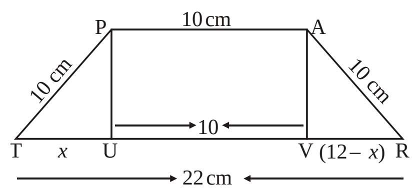
5. Let *PART* be the given trapezium

$$PA = 10\text{cm}, TR = 22\text{cm}$$

$$PA = UV = 10\text{cm}$$

Let PU be h cm and UT be x cm

$$RV = (12-x)\text{ cm}$$



In ΔPTU ;

$$PU^2 = PT^2 - TU^2$$

$$PU^2 = (10)^2 - x^2 = 100 - x^2 \quad \ldots(1)$$

In ΔAVR ; $AV^2 = AR^2 - VR^2 = (10)^2 - (12-x)^2$

$$= 100 - 144 - x^2 + 24x$$

$$= -44 - x^2 + 24x \quad \ldots(2)$$

From equation (1) and (2) we get

$$100 - x^2 = -44 - x^2 + 24x$$

$$100 + 44 = x^2 - x^2 + 24x$$

$$\frac{144}{24} = x$$

$$6 = x$$

Height $= 6\text{ cm}$

ΔPUT ; $TU = 6\text{cm}^2$

$$PU^2 = PT^2 - TU^2$$

$$= (10)^2 - (6)^2 = 100 - 36 = 64$$

$$PU = \sqrt{64} = 8\text{cm}$$

$$\text{Height of } = RV = 8\text{cm}$$

$$\text{Area of } PART = \frac{1}{2}(PA + TR) \times \text{height}$$

$$= \frac{1}{2}(10 + 22) \times 8\text{cm}^2$$

$$= \frac{1}{2} \times 32 \times 8\text{cm}^2 = 128\text{cm}^2$$

6. Area of the table as trapezium shape $= \frac{1}{2} \times (a+b) \times h$

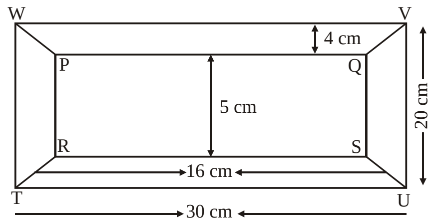
$$= \frac{1}{2} \times (1.5+2) \times 1.2 \text{ m}^2$$

$$= \frac{1}{2} \times 3.5 \times 1.2 \text{ m}^2$$

$$= 2.1 \text{ m}^2$$

Area of table $= 2.1 \text{m}^2$

7. *PQWV* and *RSTU* trapezium



In *PQWV*, $PQ = 16\text{ cm}$, $WV = 30\text{ cm}$

Area of $PQWV = \frac{1}{2}(PQ + WV) \times h$

$$= \frac{1}{2}(16+30) \times 4 \text{ cm}^2$$

$$= \frac{1}{2} \times 46 \times 4 \text{ cm}^2 = 92 \text{ cm}^2$$

Area of $RSTU = \frac{1}{2}(RS + TU) \times h$

$$= \frac{1}{2}(16+30) \times 4 \text{ cm}^2 = 92 \text{cm}^2$$

PRTW and *QSVU* trapezium

In *PRTW*,

$PR = 5\text{cm}$, $WT = 20\text{cm}$, Height $= \frac{30-16}{2} = 7\text{ cm}$

Area of $PRTW = \frac{1}{2}(PR + WT) \times h = \frac{1}{2}(5+20) \times 7\text{cm}^2 = 87.5\text{cm}^2$

Area of $QSUV = \frac{1}{2}(QS + UV) \times h$

$$= \frac{1}{2} \times (5+20) \times 7 = \frac{1}{2} \times 25 \times 7 = 87.5 \text{ cm}^2$$

PQRS rectangular.

$PQ = 16\text{ cm}$; $PR = 5\text{ cm}$

Area of $PQRS = PQ \times PR$

$$= 16 \times 5 \text{cm}^2 = 80 \text{cm}^2$$

Area of given figure.

$= \text{Area of } (PQWV + RSTU + PRTW + QSVU + PQRS)$

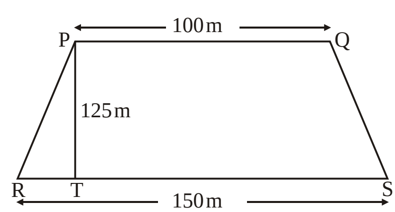
$= (92 + 92 + 87.5 + 87.5 + 80) \text{ cm}^2 = 459 \text{cm}^2$

8. Area of *PQRS* ;

$PQ = 100\text{ m}$,

$RS = 150\text{ m}$,

$PT = 125\text{ m}$.



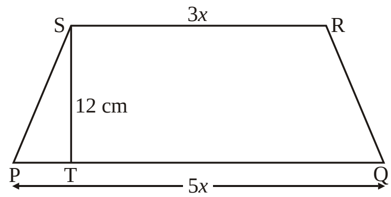
Area of $PQRS = \frac{1}{2} \times (PQ + RS) \times PT$

$$= \frac{1}{2}(100+150) \times 125 \text{ m}^2$$

$$= 125 \times 125 \text{ m}^2$$

$$= 15625 \text{ m}^2$$

9. Area of trapezium $= 720\,\text{cm}^2$



Area of $PQRS = \frac{1}{2}(PQ + SR) \times ST$

$$720 = \frac{1}{2}(5x + 3x) \times 12$$

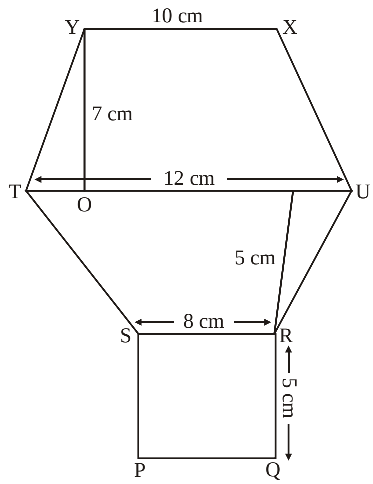
$$720 = \frac{8x}{2} \times 12 \Rightarrow 720 = 48x$$

$\Rightarrow \quad x = 15\text{cm}$

Length of $SR = 3 \times 15 = 45\text{cm}$

Length of $PQ = 5 \times 15 = 75\,\text{cm}$

10. *UTXY* Trapeziums,



Area of $UTXY = \frac{1}{2}(XY + TU) \times YO$

$$UTXY = \frac{1}{2}(10 + 12) \times 7\text{cm}^2$$

$$= 77\text{cm}^2$$

TURS Traperium

Area of $TURS = \frac{1}{2}(TU + SR)UV$

$$= \frac{1}{2}(12 + 8) \times 5\,\text{cm}^2$$

$$= 50\,\text{cm}^2$$

SRPQ Rectangle.

Area of $SRPQ = SR \times RQ$

$$= 8 \times 5\,\text{cm}^2$$

$$= 40\,\text{cm}^2$$

Area of this figure $=$ Area of trapezium *TURS* and *UTXY* + Rectangle.

$$= 77 + 50 + 40\,\text{cm}^2$$

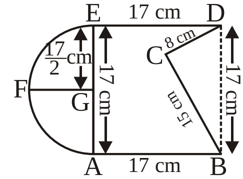
$$= 167\,\text{cm}^2$$

Exercise 11.3

1. (a) We divide given fig. into three parts as semi-circle *AFE*, square *ABDE*, and a triangle *BCD*.

In ΔBDC, we have, $a = 17$ cm, $b = 8$ cm, $c = 15$ cm.

$$S = \frac{a + b + c}{2} = \frac{17 + 8 + 15}{2} = \frac{40}{2} = 20\,\text{cm}$$



$\therefore$ Area of $\Delta BDC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{20 \times (20-17)(20-8) \times (20-15)}$$

$$= \sqrt{20 \times 3 \times 12 \times 5}$$

$$= \sqrt{4 \times 4 \times 5 \times 5 \times 3 \times 3} = 60\,\text{cm}^2$$

Area of square $ABDE = (\text{side})^2 = (17)^2 = 289\,\text{cm}^2$

Area of semi-circle $AFE = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{17}{2}\right)^2$

$$= \frac{11}{7} \times \frac{289}{4}$$

$$= \frac{3179}{28} = 113.54$$

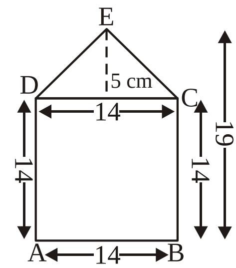
$\therefore$ Area of enclosed figure

= Area of semi-circle + Area of square – Area of triangle *BDC*

$= 113.54 + 289 - 60 = 342.54 \text{ cm}^2$.

(b) Area of square $ABCD = (\text{side})^2$

$= (14)^2 = 196 \text{ cm}^2$



Area of $\Delta CDE = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 14 \times 5 = 35 \text{ cm}^2$

$\therefore$ Area of enclosed figure

$=$ Area of square + Area of triangle

$= 196 \text{ cm}^2 + 35 \text{ cm}^2$

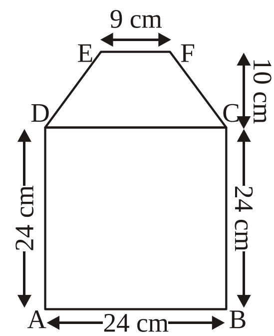
$= 231 \text{ cm}^2$

(c) Let *ABCD* be a square and *CDEF* be a trapezium.

Now, Area of square $ABCD = (\text{side})^2$

$= (24 \text{ cm})^2$

$= 576 \text{ cm}^2$



Area of trapezium *CDEF*

$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude}$

$= \frac{1}{2} \times (24 + 9) \times 10$

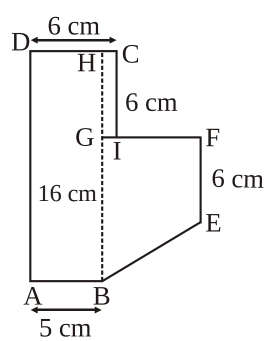
$= 33 \times 5 = 165 \text{ cm}^2$

$\therefore$ Area of closed figure $ABCDEF = 576 + 165 = 741 \text{ cm}^2$.

(d) Given Fig. can be divided into two parts, rectangle *ABCD* and trapezium *BEFG*.

Area of rectangle $ABCD = l \times b = 16 \times 5 = 80 \text{ cm}^2$

Area of trapezium *BEFG*



$= \frac{1}{2} \times (\text{sum of parllel sides}) \times \text{altitude}$

$= \frac{1}{2} \times (10 + 6) \times 7$

$= \frac{1}{2} \times 16 \times 7$

$= 8 \times 7 = 56 \text{ cm}^2$.

Area of rectangle $CIGH = l \times b = 6 \times 1 = 6 \text{ cm}^2$

$\therefore$ Area of enclosed Fig. *ABEFGHDA*

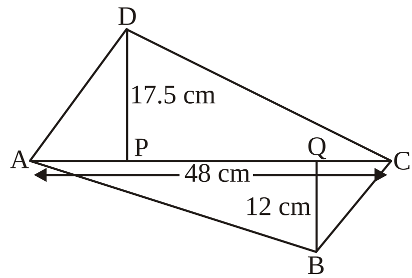
$=$ Area of rectangle *ABCD* + Area of rectangle *CIGH* + Area of trapezium

$= 80 + 56 + 6 = 142 \text{ cm}^2$.

2. Given : In quadrilateral *ABCD*,

Diagonal $AC = 48 \text{ cm}$, $DP = 17.5 \text{ cm}$, $BQ = 12 \text{ cm}$

Area of Quadrilateral *ABCD*



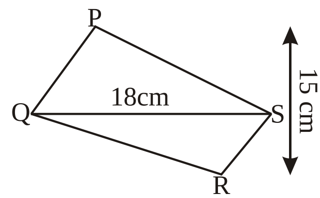
$$= \frac{1}{2} \times d \times (h_1 + h_2)$$

$$= \frac{1}{2} \times 48 \times (17.5 + 12)$$

$$= 24 \times 29.5$$

$$= 708\,\text{cm}^2.$$

3. Area of quadrilateral *PQRS*



$$= \frac{1}{2} \times d \times (h_1 + h_2)$$

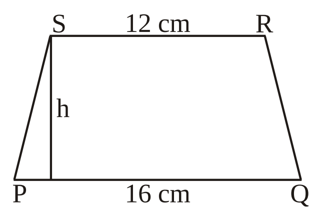
$$= \frac{1}{2} \times \text{diagonal} \times (\text{sum of perpendiculars})$$

$$= \frac{1}{2} \times 18 \times 15 = 9 \times 15 = 135\,\text{cm}^2.$$

4. Given : Area of swimming pool in the shape of a trapezium = 112 m^2

$PQ = 16\,\text{m}$, $SR = 12\,\text{m}$, $h = ?$

$\therefore$ Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides) $\times$ height (depth)



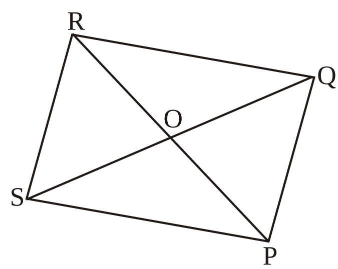
$$112 = \frac{1}{2} \times (16 + 12) \times h$$

$$112 = \frac{1}{2} \times 28 \times h$$

$$\Rightarrow \quad 112 = 14 \times h$$

$$\therefore \quad h = \frac{112}{14} = 8\,\text{m}.$$

5. Given : diagonals $SQ = 30\,\text{cm}$, $PR = 18\,\text{cm}$.



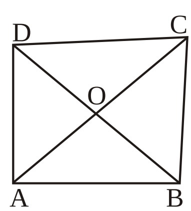
$\therefore$ Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$$= \frac{1}{2} \times 30 \times 18$$

$$= 15 \times 18$$

$$= 270\,\text{cm}^2.$$

6. Given : Area of rhombus = 216 cm^2, $d_1 = 18\,\text{cm}$, $d_2 = ?$



$$\text{Area of rhombus} = \frac{1}{2} \times 18 \times d_2$$

$$216 = 9 \times d_2$$

$$\Rightarrow \quad d_2 = \frac{216}{9} = 24\,\text{cm}.$$

7. Area of one tile $= \frac{1}{2} \times d_1 \times d_2$

$$= \frac{1}{2} \times 20 \times 28$$

$$= 280\,\text{cm}^2$$

$$= \frac{280}{100 \times 100}\,\text{m}^2 \qquad \left[\because 1\text{cm}^2 = \frac{1}{100 \times 100}\,\text{m}^2\right]$$

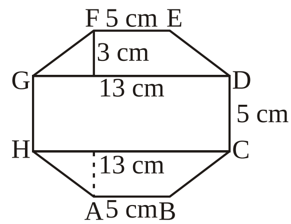
Area of 2550 tiles $= \dfrac{280}{100 \times 100} \times 2550 = \dfrac{28 \times 255}{100}\ \text{m}^2$

Now, cost of polishing the floor of 1 m^2 area = ₹ 25

$\therefore$ Cost of polishing the floor of $\dfrac{28 \times 255}{100}\ \text{m}^2$ Area = ₹ $\dfrac{\overset{7}{\cancel{28}} \times 255 \times \overset{1}{\cancel{25}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{100}}}$

$= 7 \times 255 \times 1 =$ ₹ $1785.$

8. We divide the table into three parts.
trapezium *ABCH*,
rectangle *CDGH*
and another trapezium *DEFG*.



Now, Area of trapezium *ABCH*

$= \frac{1}{2} \times$ (sum of parallel sides) × height

$= \frac{1}{2} \times (13 + 5) \times 3 = \frac{1}{2} \times 18 \times 3$

$= 9 \times 3 = 27\ \text{cm}^2.$

Area of trapezium *DEFG* $= \frac{1}{2} \times$ (sum of parallel sides) × height

$= \frac{1}{2} \times (13 + 5) \times 3 = \frac{1}{2} \times 18 \times 3 = 9 \times 3 = 27\ \text{cm}^2$

Area of rectangle $CDGH = l \times b$

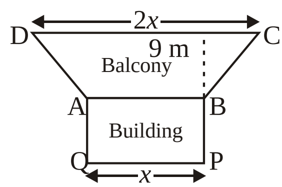
$= 13 \times 5 = 65\ \text{cm}^2$

Area of the regular octagon = Area of *ABCH* + Area of *CDGH* + Area of *DEFG*

$= 27 + 65 + 27 = 119\ \text{cm}^2.$

9. Let *PQAB* be a building and *ABCD* is a balcony attached to the building in the shape of a trapezium.

Area of trapezium $= \frac{1}{2} \times$ (sum of it's parallel sides) × height



$\Rightarrow \quad 18 = \frac{1}{2} \times (x + 2x) \times 9$

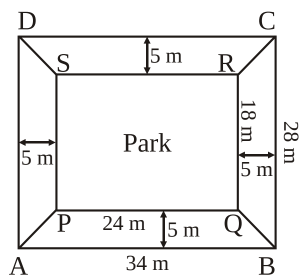
$\Rightarrow \quad 18 \times 2 = 3x \times 9$

$\Rightarrow \quad x = \dfrac{\overset{2}{18} \times 2}{3 \times \underset{1}{9}} = \dfrac{4}{3}$

$\therefore$ length of the side of the balcony not attached to the building $= 2x$

$= 2 \times \frac{4}{3} = \frac{8}{3} = 2.666 = 2.67\ \text{m}$

10. Area of trapezium type park *ABQP*



$= \frac{1}{2} \times$ (sum of parallel sides) × h

$= \frac{1}{2} \times (34 + 24) \times 5$

$= \frac{1}{2} \times 58 \times 5 = 29 \times 5 = 145\ \text{m}^2.$

Area of another trapezium type park $BCRQ = \frac{1}{2} \times$ (sum of parallel sides) $\times h$

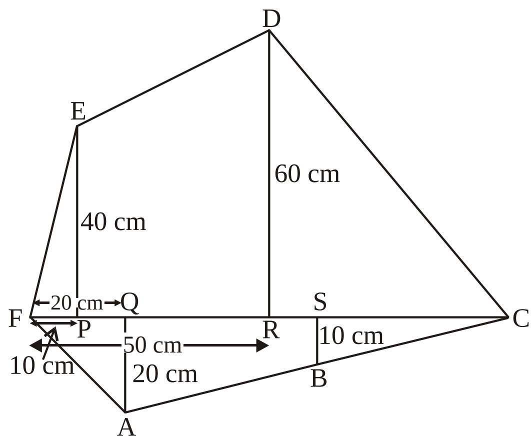
$$= \frac{1}{2} \times (28+18) \times 5$$

$$= \frac{1}{2} \times 46 \times 5 = 23 \times 5 = 115\,\text{m}^2.$$

Similarly, Area of Trapezium $CDSR$ = Area of $ABQP$ $= 145\,\text{m}^2$

And area of Trapezium $ADSP$ = Area of $BCRQ = 115\,\text{m}^2$.

11. **Given :** $FP = 10\,\text{cm}$, $FQ = 20\,\text{cm}$, $FR = 50\,\text{cm}$, $FS = 60\,\text{cm}$, $FC = 100\,\text{cm}$.



Area of $\Delta FQA = \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times FQ \times AQ$$

$$= \frac{1}{2} \times 20 \times 20$$

$$= 200\,\text{cm}^2$$

Area of trapezium $ABSQ$

$$= \frac{1}{2} \times h \times \text{(sum of parallel sides)}$$

$$= \frac{1}{2} \times SQ \times (AQ + BS)$$

$$= \frac{1}{2} \times 40 \times (20 + 10)$$

$$= 20 \times 30 = 600\,\text{cm}^2$$

Area of $\Delta BCS = \frac{1}{2} \times$ base height $= \frac{1}{2} \times CS \times BS = \frac{1}{2} \times 40 \times 10 = 20 \times 10 = 200\,\text{cm}^2$

Area of $\Delta CDR = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times RC \times RD$

$$= \frac{1}{2} \times 50 \times 60 = 25 \times 60 = 1500\,\text{cm}^2$$

Area of trapezium $PRDE = \frac{1}{2} \times h \times$ (sum of parallel sides)

$$= \frac{1}{2} \times 40 \times (40 + 60) = \frac{1}{2} \times 40 \times 100$$

$$= 20 \times 100 = 2000\,\text{cm}^2$$

Area of $\Delta FEP = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times FP \times EP = \frac{1}{2} \times 10 \times 40 = 10 \times 20 = 200\,\text{cm}^2$

$\therefore$ Area of the polygon $ABCDEF$

= Area of ΔFQA + Area of trapezium $ABSQ$ + Area of ΔBCS + Area of ΔCDR + Area of trapezium $PRDE$ + Area of ΔFEP

$$= 200 + 600 + 200 + 1500 + 2000 + 200$$

$$= 4700\,\text{cm}^2.$$

MCQs

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (b)

Surface Area and Volume

Exercise 12.1

1. Given : $l = 30\,\text{m},\ b = 25\,\text{m},\ h = 18\,\text{m}$

Total surface area of the rectangular box $= 2(lb + bh + lh)$

$$= 2\,(30 \times 25 + 25 \times 18 + 30 \times 18)$$
$$= 2(750 + 450 + 540)$$
$$= 2 \times 1740 = 3480\,\text{m}^2$$

∵ Cost of painting its outer surface of 1 m^2 = ` 12

∴ Cost of painting its outer surface of 3480 $\text{m}^2 = 12 \times 3480 =$ ` 41,760.

2. Since, only four walls are to be white washing, we need to find only the lateral surface area of the hall.

Given : $l = 150\,\text{m},\ b = 25\,\text{m},\ h = 6\,\text{m}$.

Lateral surface area or area of four wall $= 2 \times h\,(l + b) = 2 \times 6 \times (150 + 25)$

$$= 12 \times 175 = 2100\,\text{m}^2$$

Area of roof $= l \times b = 150 \times 25 = 3750\,\text{m}^2$

Total area to be white washing $= 3750 + 2100 = 5850\,\text{m}^2$

∵ The cost of white washing its four walls and roof of 1 m^2 = ` 20

∴ The cost of white washing its four walls and roof of 5850 m^2 = ` (20×5850)

= ` 117000

Area of floor $= l \times b = 150 \times 25 = 3750\,\text{m}^2$

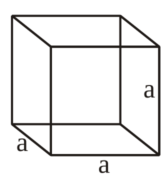
∵ Cost of polishing the floor of 1 m^2 Area = ` 40

∴ Cost of polishing the floor of 3750 m^2 Area of = ` $(40 \times 3750)$ = ` 1,50,000.

3. Let the side of cube be 'a' m.

Given : Total surface area $= 3750\,\text{m}^2$

Total surface area of cube $= 6a^2$



$$\Rightarrow \quad 6a^2 = 3750$$

$$\Rightarrow \quad a^2 = \frac{\overset{625}{\cancel{3750}}}{\underset{1}{\cancel{6}}} = 625$$

$$\Rightarrow \quad a = \sqrt{625} = 25\,\text{m}.$$

∴ side of cube is 25 m.

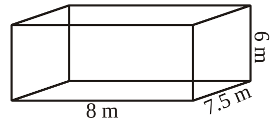
4. Since, Harshit painted the outer surface of the cuboidal box, we need to find out its total surface area.

Given : $l = 8\,\text{m},\ b = 7.5\ \text{m},\ h = 6\,\text{m}$

Total surface area of the cuboidal box $= 2\,(lb + bh + lh)$

$$= 2\,(8 \times 7.5 + 7.5 \times 6 + 8 \times 6)$$
$$= 2\,(60 + 45 + 48)$$
$$= 2 \times 153$$
$$= 306\ \text{m}^2$$

Again, since Harshit did not paint the bottom of the box



$\therefore$ Area of the bottom $= l \times b = 8 \times 7.5 = 60\,\text{m}^2$.

Now, the required surface area

Painted by him $=$ Total surface area – area of the bottom

$= 306 - 60 = 246\,\text{m}^2$

5. Total surface Area of first box i.e. cuboidal box $= 2(lb + bh + hl)$

$= 2(70 \times 50 + 50 \times 60 + 60 \times 70)$

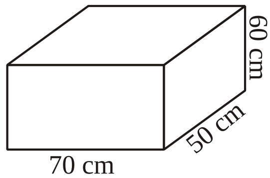
$= 2 \times 10700 = 21400\,\text{cm}^2$

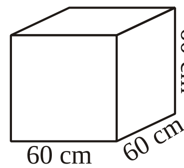
Total surface area of 2nd box

i.e., cubical box $= 6a^2$

$= 6 \times (60)^2 = 6 \times 3600$

$= 21600\,\text{cm}^2$

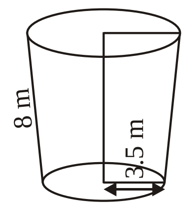




Since $21600\,\text{cm}^2 > 21400\,\text{cm}^2$

$\therefore$ Cubical box requires more material to make.

6. **Given :** $h = 8\,\text{m}$, $r = 3.5$ m



Total surface area $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 3.5 \times (8 + 3.5)$

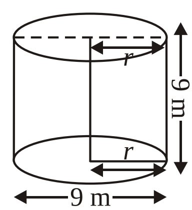
$= \frac{154}{7} \times 11.5 = 22 \times 11.5 = 253\ \text{m}^2$

$\because$ Cost of the metal sheet of 1 m^2 Area = ₹ 130

$\therefore$ Cost of the metal sheet of 253 m^2 Area = ₹ (130×253) = ₹ 32,890.

7. First is the cylindrical box and the second is cubical box.

$h = 9\,\text{cm}$, $D = 9\,\text{cm}$



$\therefore$ $r = \frac{D}{2} = \frac{9}{2}$ cm

Lateral surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{9}{2} \times 9$

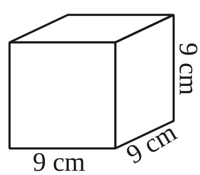
$= \frac{1782}{7} = 254.57$

Sides of cube = $l = b = h = 9\,\text{cm}$ i.e., $a = 9\,\text{cm}$

Lateral surface area $= 4a^2$

$= 4 \times (9)^2$

$= 4 \times 81 = 324$



It is clear from the above that lateral surface Area of both figure is not same.

8. **Given :** $l = 50\,\text{cm}$, $b = 35\,\text{cm}$, $h = 10\,\text{cm}$

Total surface area of chocolate box $= 2(lb + bh + hl)$

$= 2(50 \times 35 + 35 \times 10 + 10 \times 50)$

$= 2 \times 2600 = 5200\,\text{cm}^2$

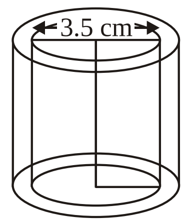
Since, 1 box requires the wrapper to be covered equal to the total surface Area of chocolate box.

$\therefore$ 1 chocolate box requires wrapper $= 5200\,\text{cm}^2$

$\therefore$ 60 such chocolate box requires wrapper $= 60 \times 5200\,\text{cm}^2$

$$= \frac{60 \times 5200}{100 \times 100}\,\text{m}^2 = 31.2\,\text{m}^2$$

9. **Given :** Inner diameter of circular well = 3.5 m



$\therefore$ Inner radius of circular well, $r = \frac{D}{2} = \frac{3.5}{2}$ m

depth i.e., height of the well = 15 m

Inner curved surface area of well $= 2\pi rh$

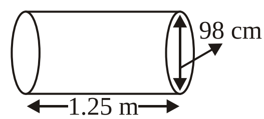
$$= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 15\,\text{m}^2$$

$$= \frac{1155}{7} = 165\,\text{m}^2$$

$\because$ Cost of plastering the inner curved surface area of 1 m^2 = ₹ 25

$\therefore$ Cost of plastering the inner curved surface area of 165 m^2 = ₹ $25 \times 165 =$ ₹ 4125

10. **Given :** $D = 98\,\text{cm}$



road roller

$$\therefore \quad r = \frac{D}{2} = \frac{98}{2} = 49\,\text{cm},$$

$$= 0.49\ \text{m}$$

$$h = 1.25\,\text{m}$$

Area covered in 1 revolution

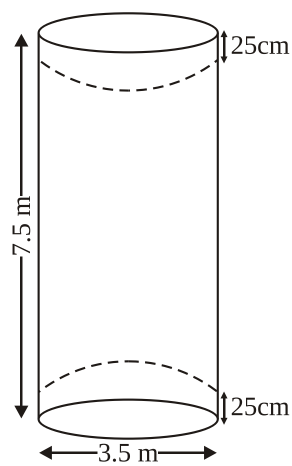
$= $ curved surface area of the cylindrical wheel

$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.49 \times 1.25\,\text{m}^2$$

$$= 3.85\ \text{m}^2$$

$\therefore$ Area covered in 900 revolution $= 900 \times 3.85 = 3465\,\text{m}^2$.

11. Height of cylindrical pillar = 7.5 m



$\because$ Diameter of circular surface = 3.5 m

$\therefore$ Radius of circular surface $= \frac{3.5}{2} = 1.75$ m

Covered height = 25 cm + 25 cm

= 50 cm = 0.5 m.

Remaining height = 7.5 m – 0.5 m

= 7 m.

Then, the area of the pillar which is to be painted

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 1.75 \times 7$$

$$= 77\ \text{m}^2$$

12. **Given :** $l = 7\,\text{m}$, $b = 6\,\text{m}$, $h = 4\,\text{m}$

Area of the doors + windows = 7 m^2

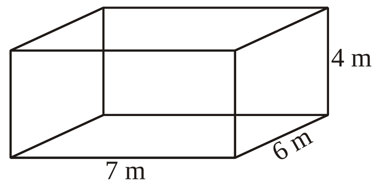
Area of 4 walls of the classroom $= 2(l + b) \times h$

$$= 2 \times (7 + 6) \times 4$$

$$= 2 \times 13 \times 4$$

$$= 104\,\text{m}^2$$

But, the doors and windows occupy an area of 7 m^2.



$\therefore$ Area of the walls $= 104\ m^2 - 7\ m^2$

$= 97\ m^2$

Area of the roof $= l \times b$

$= 7 \times 6 = 42\ m^2$

Total area to be whitewashed $= 97\ m^2 + 42\ m^2 = 139\ m^2$

Cost of whitewashing of 1 m^2 = ` 15

$\therefore$ Cost of whitewashing of 139 m^2 = ` 15 × 139 = ` 2085

Exercise 12.2

1. (a) Volume of a cube $= (\text{side})^3 = (15)^3\ cm^3 = 3375\ cm^3$

(b) Value of a cube $= (\text{side})^3 = (9.5)^3\ cm^3 = 857.375\ cm^3$

2. (a) Volume of a cuboid $= lbh \quad \Rightarrow \quad 30 \times 15 \times 12\ cm^3 = 5400\ cm^3$

(b) Volume of a cuboid $= lbh$

$\Rightarrow \quad 150\ cm \times 95\ cm \times 0.5\ cm \quad \Rightarrow \quad 7125\ cm^3 = 0.007125\ m^2$

3. (a) $r = 7\ cm$, $h = 40\ cm$

Total surface area $= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 7 \times (40 + 7)$

$= 44 \times 47 = 2068\ cm^2$.

Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 40 = 1760\ cm^2$

Volume $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 40 = 6160\ cm^2$

(b) $r = 2.8m$, h = 1.5 m

Volume $= \pi r^2 h = \frac{22}{7} \times 2.8 \times 2.8 \times 1.5 = 36.96\ m^3$

Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 2.8 \times 1.5 = 26.4\ m^2$

Total surface area $= 2\pi r\,(r + h)$

$= 2 \times \frac{22}{7} \times 2.8\,(1.5 + 2.8) = 75.68 m^2$

4. Length of a cuboidal shape gold biscuit $= 8cm$

breadth of a cuboidal shape gold biscuit $= 5cm$

height of a cuboidal shape gold biscuit $= 2cm$

Volume of gold biscuit $= l \times b \times h$

$= 8 \times 5 \times 2\ cm^3$

$= 80\ cm^3$

Volume of small lockets $= 2.5\ cm^3$

Number of locker made form this gold biscuit

$= \frac{80}{2.5} = 32$

Thus, 32 lockets can be made form this gold biscuit.

5. Let, length of the side of cube II $= x \text{ cm}$

Volume of the side of cube II $= x^3 \text{cm}^3$

So, length of the side of cube I $= 2 \times x = 2x \text{ cm}$

Volume of the side of cube I $= (2x)^3 \text{ cm}^3$

$= 8x^3 \text{cm}^3$

Ratio of volume of the side of cube I and cube II

$= 8x^3 : x^3$

$= 8 : 1$

6. In external part ;

$l = 36 \text{ cm}$; $b = 25 \text{ cm}$; $h = 16.5 \text{ cm}$

Volume of External aluminium box $= 36 \times 25 \times 16.5 \text{ cm}^3$

$= 14850 \text{ cm}^3$

In internal part ;

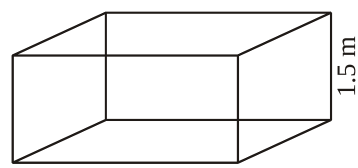
$l = 36 - 1.5 \times 2 = 33 \text{ cm}$

$b = 25 - 1.5 \times 2 = 22 \text{ cm}$

$h = 16.5 - 1.5 = 15 \text{ cm}$

Volume of internal aluminum box $= 33 \times 22 \times 15 \text{ cm}^3$

$= 10890 \text{cm}^3$



volume of aluminium $= 14850 - 10890 \text{cm}^3 = 3960 \text{ cm}^3$

weight of aluminium $= 3960 \times 4.5$ grams.

$= 17820$ grams or 17.820 kg

7. Size of alarm clock

$l = 5 \text{ cm}$; $h = 10 \text{ cm}$; $b = 10 \text{ cm}$

Volume of alarm clock $= l \times h \times h$

$= 5 \times 10 \times 10 \text{ cm}^3 = 500 \text{ cm}^3$

Size of packing box.

$l = 1$; $b = \frac{1}{2}$ m ; $h = \frac{3}{4}$ m

or

$l = 100 \text{ cm}$, $b = 50 \text{ cm}$; $h = 75$ cm

volume of packing box $= l \times b \times h$.

$= 100 \times 50 \times 75 \text{ cm}^3$

$= 375000 \text{ cm}^3$

Number of alarm clock packed into a box of size $= \frac{375000}{500} = 750.$

8. Edges of I cube $= 18 \text{ cm}$

Volume of I cube $= (18)^3 \text{ cm}^3$

$= 5832 \text{ cm}^3$

Edges of II cube $= 24$ cm

Volume of II cube $= (24)^3 \text{ cm}^3$

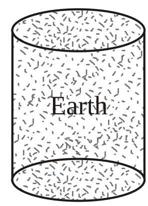
$= 13824 \text{ cm}^3$

Edges of III cube $= 30 \text{ cm}$

$$\text{Volume of III cube} = (30)^3 \text{ cm}^3$$
$$= 27000 \text{ cm}^3$$
$$\text{Total volume of three cubes} = (5832 + 13824 + 27000) \text{ cm}^3$$
$$= 46656 \text{ cm}^3$$
$$\text{Edge of the new cube} = \sqrt[3]{46656} \text{ cm}^3$$
$$= 36 \text{ cm}$$

9. **Given :** 594 m^3 earth dug out means volume of cylinder i.e. (well)

$\therefore V = 594 \text{ m}^3,\ d = 6 \text{ m},\ r = \frac{d}{2} = \frac{6}{2} = 3 \text{ m},\ h = ?$



$$\text{Volume of dug out} = \pi r^2 h$$
$$594 = \frac{22}{7} \times 3 \times 3 \times h$$
$$h = \frac{\overset{27}{\cancel{594}} \times 7}{\underset{1}{\cancel{22}} \times 9} = 21 \text{ m.}$$

$\therefore$ The depth of the well is 21 m.

10. **Given :**
$$\text{CSA of cylinder} = 4400 \text{ cm}^2$$
$$\text{Circumference of its base} = 220 \text{ cm}$$
$$\text{Volume of the cylinder} = ?$$

$\because$ $\text{CSA} = 4400$

$\Rightarrow$ $2\pi rh = 4400$

$\Rightarrow$ $2 \times \frac{22}{7} \times r \times h = 4400$

$\Rightarrow$ $r \times h = 100 \times 7 = 700$ ...(1)

Again, circumference of base $= 220$

$\Rightarrow$ $2\pi r = 220$

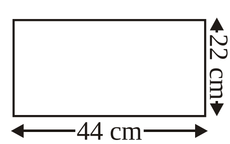
$\Rightarrow$ $2 \times \frac{22}{7} \times r = 220$

$\Rightarrow$ $r = \frac{220 \times 7}{2 \times 22} = 35 \text{ cm}$ ...(2)

From (1) and (2), we get
$$35 \times h = 700$$
$$h = \frac{700}{35} = 20 \text{ cm.}$$

$\therefore$ $\text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 35 \times 35 \times 20$
$$= 22 \times 5 \times 700 = 77000 \text{ cm}^3.$$

11. Height of cylinder $(h) = 22 \text{ cm}$



Let the radius of cylinder be r cm.

Circumference of base of cylinder = 44 cm.

[$\because$ length of rectangle become the circumference of base]

$\Rightarrow$ $2\pi r = 44$

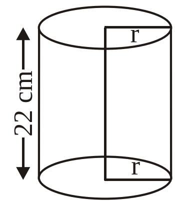
$\Rightarrow$ $2 \times \frac{22}{7} \times r = 44$

$\Rightarrow \quad r = \frac{7 \times 44}{2 \times 22} = 7\,\text{cm}$

$\therefore$ Total surface Area $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (22 + 7)\,\text{cm}^2$

$= 44 \times 29\,\text{cm}^2 = 1276\,\text{cm}^2$



Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 22$

$= 22 \times 7 \times 22 = 3388\,\text{cm}^3$.

12. **Given :** Dimensions of hall are $l = 150\,\text{m}$, $b = 85\,\text{m}$, $h = 12\,\text{m}$

$\therefore$ Volume of the hall $= l \times b \times h$

$= 150 \times 85 \times 12\,\text{m}^3 = 153000\,\text{m}^3$

Volume of air required by each (i.e. 1) person

$= 50\,\text{m}^3$

$\therefore$ No. of required persons $= \frac{\text{Volume of the hall}}{\text{Volume of air requires by each person}}$

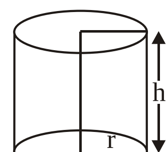
$= \frac{150 \times 85 \times 12}{50} = 3060.$

13. Diameter of well $= 7$ m

$r = \frac{d}{2} = \frac{7}{2}\,\text{m}$

Height (depth) $h = 20\,\text{m}$

Volume of earth dug out $=$ Volume of rectangular plot



well

$\Rightarrow \quad \pi r^2 h = l \times b \times h$

$\Rightarrow \quad \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 14 \times 11 \times h$

$\Rightarrow \quad h = \frac{11 \times 10 \times 7}{14 \times 11} = 5\,\text{m}$

Volume of earth dug out $= \pi r^2 h. = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 770\,\text{m}^3$

14. **Given :** l of brick = 25 cm, b of brick = 10.5 cm, h of brick = 9 cm.
l of wall = 18 m, b of wall = 8 m, h of wall = 21 cm

$\therefore$ Number of bricks $= \frac{18 \times 100 \times 8 \times 100 \times 21}{25 \times 10.5 \times 9}$

$= 2 \times 4 \times 8 \times 100 \times 2 = 12800.$

15. Volume of 1 cube $= (\text{side})^3 = (6)^3 = 216\,\text{cm}^3$

$\therefore$ Volume of 6 cubes $= 6 \times 216 = 1296\,\text{cm}^3$

$\therefore$ Volume of new solid $=$ Volume of 6 cubes $= 1296\,\text{cm}^3$

16. Let the number be coins be x.
$r = 0.75\,\text{cm}$, $h = 0.2\,\text{cm}$,

h of cylinder $= 8\,\text{cm}$

d of cylinder $= 6$ cm,

r of cylinder $= \frac{d}{2} = 3\,\text{cm}.$

$$\text{Volume of right circular cylinder} = \pi r^2 h = \frac{22}{7} \times (3)^2 \times 8$$

$$\text{Volume of 1 coin} = \frac{22}{7} \times 0.75 \times 0.75 \times 0.75 \times 0.2$$

$$\therefore \quad \text{No. of coins} = \frac{\text{Volume of right circular cylinder}}{\text{Volume of 1 coin}}$$

$$= \frac{\frac{22}{7} \times 3 \times 3 \times 8}{\frac{22}{7} \times 0.75 \times 0.75 \times 0.2} = 640.$$

17. Dimension of the water tank = 10 m × 7.5 m × 4 m

$\therefore$ Volume of the tank $= l \times b \times h$

$= (10 \times 7.5 \times 4)\,\text{m}^3 = 300\ \text{m}^3$

Then,

$\because$ Capacity of the tank that has 1 m^3 volume

$= 1000\ l$

$\therefore$ Capacity of the tank that has 300 m^3 volume

$= 300 \times 1000\,l = 300000\,l$

$\because$ Time taken to pump 400*l* of water = 1 min

$\therefore$ Time taken to pump 300000*l* of water $= \frac{1}{400} \times 300000$

$= 750\ \text{min}$

$= \frac{750}{60}\ \text{hr} = \frac{75}{6}\ \text{hr} = \frac{25}{2}\ \text{hr} = 12\frac{1}{2}\ \text{hr}$

18. Volume of water which flows out in 1 second from the pipe

= Speed of water × Area of the cross section of the pipe

Given : Area of the section of the tap $= 5\,\text{cm}^2$

$\therefore$ Volume of water $= 30 \times 5 = 150$

$\Rightarrow$ Volume of water which flows out in 1 hr (i.e., 60 min) from the pipe

$= 150 \times 60 \times 60 \quad [\because 1h = 60\,\text{min} = 60 \times 60\text{sec}]$

$= 540000\,\text{cm}^3 \quad [\because 1000\text{cm}^3 = 1l]$

$= 540\ l.$

19. Let breadth of the room $= x$ m, height of the room = 4 m

then, length of the room $= 2x$ m,

$\Rightarrow \quad 2(l + b)h = 192$

$\Rightarrow \quad 2 \times (2x + x) \times 4 = 192$

$\Rightarrow \quad x = \frac{192}{24} = 8.$

$\therefore$ Breadth $(b) = x = 8\,\text{m}$

Length $(l) = 2x = 16\,\text{m}$

Now, volume of the room $= l \times b \times h$

$= 16 \times 8 \times 4 = 512\,\text{m}^3.$

MCQs

1. (d) **2.** (b) **3.** (a) **4.** (b) **5.** (b) **6.** (c) **7.** (b) **8.** (a) **9.** (b) **10.** (d)